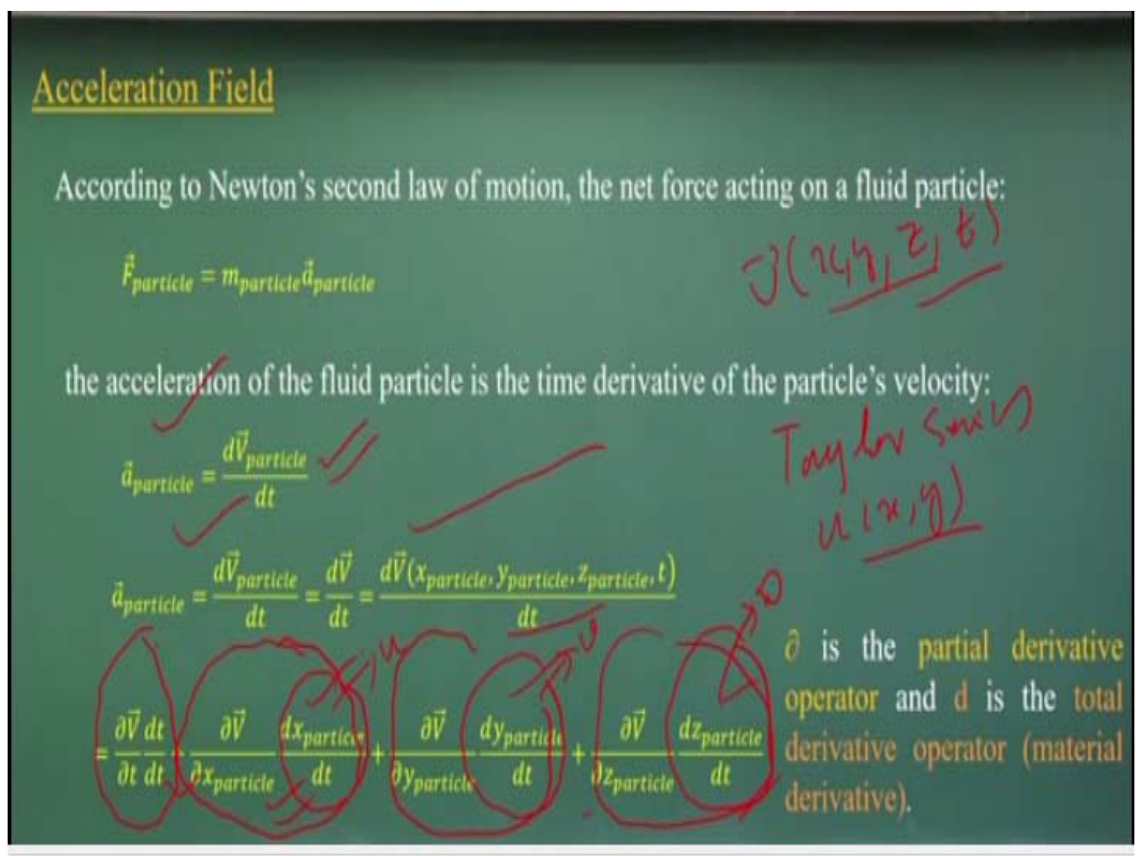

So, similar way if I am looking at Newton's second law also see that force is equal to mass into acceleration, force is vector component, acceleration is the vector component, okay and both are the parallels okay, so force and the vector, at the particle levels like in solid mechanics, the force we can put is mass into acceleration.

$$\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$$

Now, let us look it at the particle levels, if I had to find out what is the acceleration of the fluid particles?

Is nothing else is a time derivative of velocity of the particles, as you know it from class 10, 11th and 12th I am just doing this time derivative of the velocity; particle velocities with respect to time and that is what represents the accelerations, at the particles levels you will have these. Now, if you look at this the velocities has a variability in a positions and the time because of that though when you define a; derivative with respect to time you will have a local component okay, you will have a with a x particle directions, y particle directions and z particle direction.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

This is nothing else if you are considering is a 2 variables like I just discussed you the Taylor series, if you remember it defining for the 2 variables in this case, I have a Taylor series of 4 variables the x, y, z and the t. If you expand it and take it only these 4 terms, you will get this component nothing else, we are near to the same Taylor series concept what we discussed in a single variables, independent variables, how the Taylor series expansion, the 2 independent variables how the Taylor series expansion.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} = \frac{d\vec{V}}{dt} = \frac{d\vec{V}(x_{particle}, y_{particle}, z_{particle}, t)}{dt}$$

$$= \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{particle}}\frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y_{particle}}\frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z_{particle}}\frac{dz_{particle}}{dt}$$

If you do that, you will have a the same concept what is there and by the dt, you will get these component, so mathematically we are not things you just try to understand it that the same Taylor series we have applied it but the v is a having the independent the variable 3 in a space and the time that is a reason you have the length d resistance, so if we look at this way, the $\frac{dx_{particle}}{dt}$, what is that?

The velocity in the u directions, $\frac{dy_{particle}}{dt}$, what is that; the velocities in v directions, $\frac{dz_{particle}}{dt}$, the particles; the change in the z direction without or times will be the; which is the definitions of the velocity.

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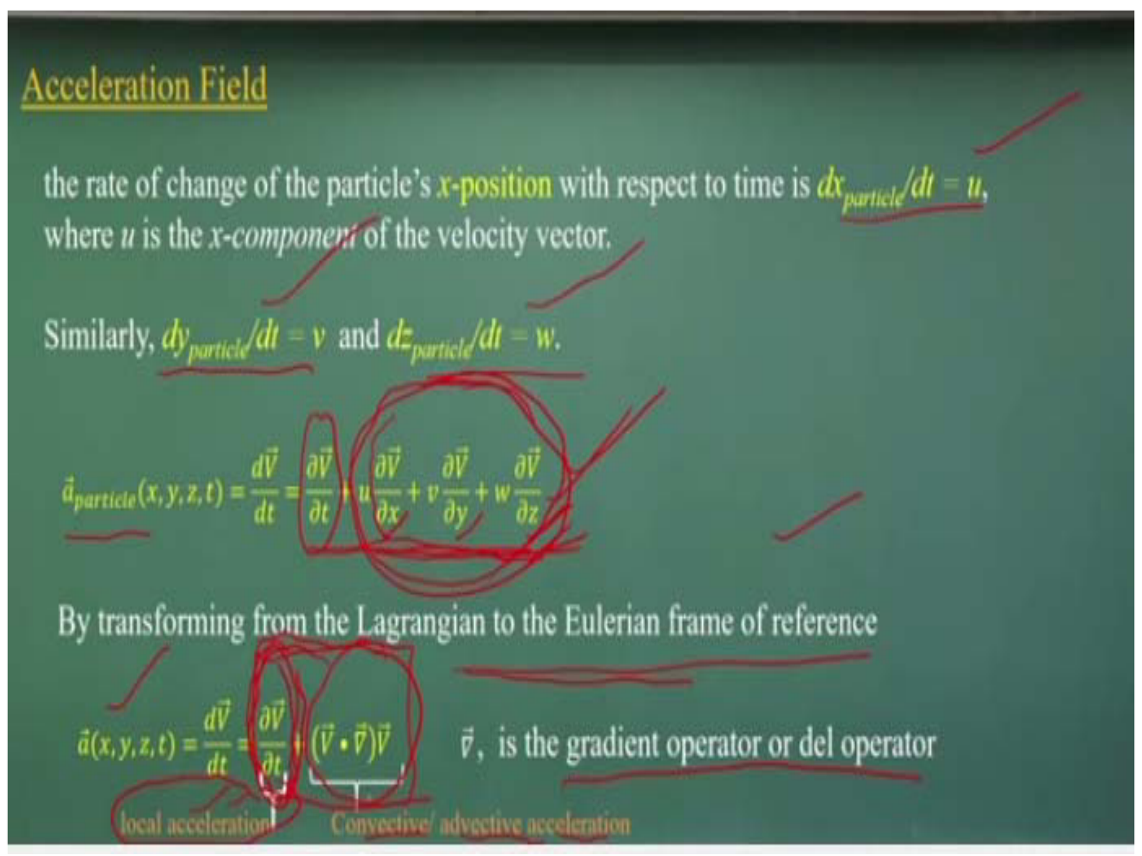

So, if I put it that I will get it the accelerations fields as I explaining it that, I will have a particle, the change of the position the particles in the x direction with respect to time will give you the velocity component in that directions, so this is for x component, so y component, z component, so the finally this accelerations will have this form, accelerations will have this form.

the rate of change of the particle's *x*-position with respect to time is $dx_{particle}/dt = u$,

where u is the *x-component* of the velocity vector.

Similarly, $dy_{particle}/dt = v$ and $dz_{particle}/dt = w$.

$$\vec{a}_{particle}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial\vec{V}}{\partial t} + u\frac{\partial\vec{V}}{\partial t} + v\frac{\partial\vec{V}}{\partial t} + w\frac{\partial\vec{V}}{\partial t}$$

So, by transferring this what do we do it actually, when you talk about either particles of large number particles or you talk about the probes, both are the same in the both frame of reference, you will get it as the same things whether you particle tracking, the probing, you will have the acceleration speed like the same at the particles and the Euler frame of reference, you are getting it these accelerations will have a 2 component.

This component and this component, if you look at expanding forms, this is what vector calibration, it is okay, it is called del operators, okay between the V and delta that is what the del; the dot product of 2 vectors okay that is what is represented this ones, if you remember it the dot product of 2 vectors, okay. So, if you look at that what is there; these the velocity is changing with respect to time and it is given is the partial derivative.

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial\vec{V}}{\partial t} + (\vec{V} \bullet \vec{\nabla})\vec{V}$$

$\vec{\nabla}$ = is the gradient operator or del operator

Convective/ advective acceleration

$$(\vec{V} \bullet \vec{\nabla})\vec{V}$$

local acceleration

$$\frac{\partial\vec{V}}{\partial t}$$

That means, with respect to time, how much of variations and when you are considering that you do not consider the variability in the x and y, z that what is called the local accelerations but this component if you look it, it is called convective or advective accelertaion that because the particles are having different velocity gradient okay, since they have a different velocity gradient, so let us look at this component of acceleration and particles or the Eulerian and Lagrangian frames, they will be the same.

If you look at that this is what is called the local accelerations, the velocity changes as with respect to the time only, so that means at the probe equations because of only the velocity changes at that locations will give us the local accelerations. Velocity is also changes because of change of the velocity field that is the velocity variances x gradient, y gradient, z gradient and the u, v, w component.

These components change of the velocities is called convective or advective accelerations component, so this is 2 components; one is local acceleration component and other is convective acceleration. In vector (()) (45:37), we can very simple way represented the local accelerations and the convective acceleration component.

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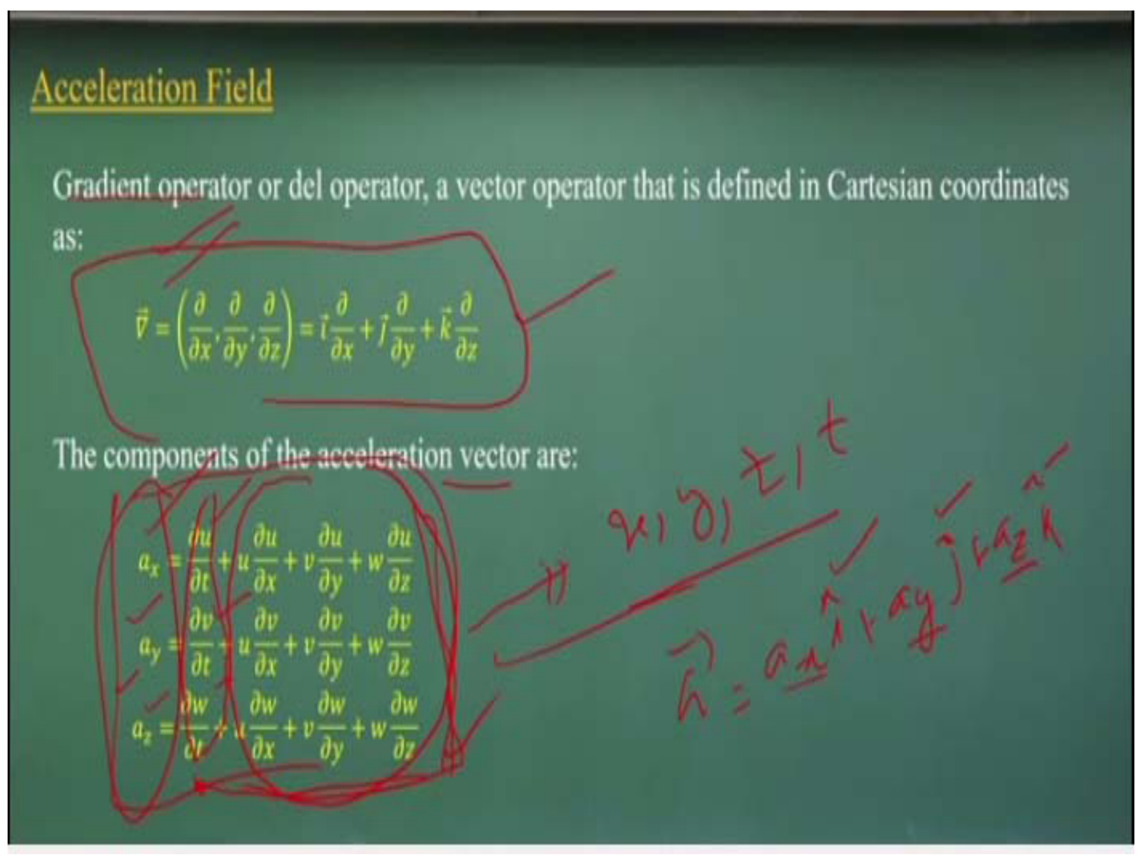

Now, if you look at these same acceleration fields, if I looking it as a delta operators or the gradient operator, you will know it we can define it as i, j functions like this

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \vec{\imath}\frac{\partial}{\partial x} + \vec{\jmath}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

and the accelerations component; the scalar component of accelerations in the x direction, y and z direction can have it like this. So, what we have done it we just put it the accelerations,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

So, this is the convective acceleration term and this is what we have local acceleration term. In a Cartesian coordinate systems, where we have a x, y, z and the t and these are showing these accelerations which will be

$$\vec{a} = a_x i + a_y j + a_z k$$

so these are the acceleration scalar components in i, j and k direction respectively. So, we can define it the accelerations component that means, this equation shows that if I know the velocity components and the velocity partial derivative components, then we can know it what is the acceleration component.

So, we can know accelerations variability in the space and the time; space and the time, if I know this velocity variability, the partial derivative of velocity with respect to the space x, y, z, also the time variability of this velocity with respect to time, if I know this component, I can compare this scalar component using this equation, you can easily remember this equations which is just a replace of u, v, w, it is nothing else representing the a_x, a_y, a_z component.

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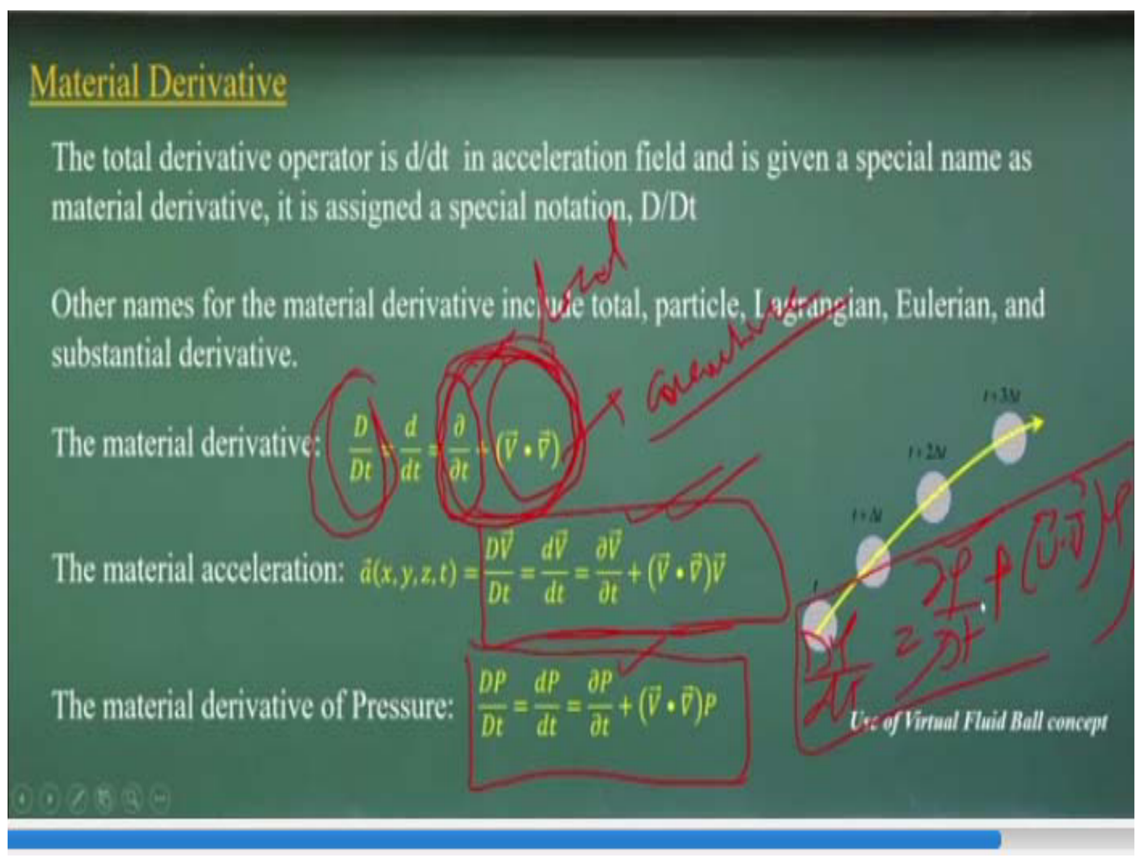

But many of the times people talk about material derivative, it is nothing else, it is that the change it is a special name of material derivative means, when you talk about a particles along the particles, you compute the derivative with respect to time is a total derivative or the material derivative or the particle derivatives is nothing else, we already proved it that these derivatives we can define as these functions that is what we have defined it, okay.

The material derivative:

$$\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{V} \bullet \vec{\nabla})$$

So, that means at the particles levels how it is varies with the time; it has a 2 component and the Eulerian frames; one is the local component and other is convective component, those the gradient we have represented like this, is a particle derivatives that means, it is a the derivative of a particles are moving it, material derivative, particle derivative all are the same, the derivative as we are getting of the velocity field and pressure field as I see that if I have a series of the boats okay, I measure how does it changes it.

That what is my total derivative compound, okay but if I am looking it in Eulerian frames which will be this local derivative and convective terms; local derivative; local and convective term this because of velocity gradients in a space domain that is what we represent the accelerations. The similar way if I talk the particles, it has some pressures, how these particles derivative changes with the time, I can define like this.

The material acceleration:

$$\vec{a}(x, y, z, t) = \frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial\vec{V}}{\partial t} + (\vec{V} \bullet \vec{\nabla})\vec{V}$$

If my particles having the density as I earlier say that particles can have the density, my virtual fluid balls can have a different density, if I define that I can define with a material derivative of the density,

$$\frac{D\rho}{Dt} = \frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + (\vec{V} \bullet \vec{\nabla})\rho$$

so if you look at that either is a density, the pressure or the velocity all the things we can define in terms of particle derivative forms or the material derivative form.

The material derivative of Pressure

$$\frac{DP}{Dt} = \frac{dP}{dt} = \frac{\partial P}{\partial t} + (\vec{V} \bullet \vec{\nabla})P$$

Some people tell different names but all are the same, we are talking about in terms of Lagrangian framework to link it with the Eulerian frameworks which is having a local and convective terms, so that is the a breaching between these derivative component between the Eulerian frameworks and the Lagrangian frameworks. The virtual fluid ball concepts and that is what is link it as I said it.

So, as this fluid having this velocity field, we can compute the accelerations field, we can compute this material derivative of the pressures, we can compute the material derivative of density with the same format, we can compute and we can solve the problems in this way.

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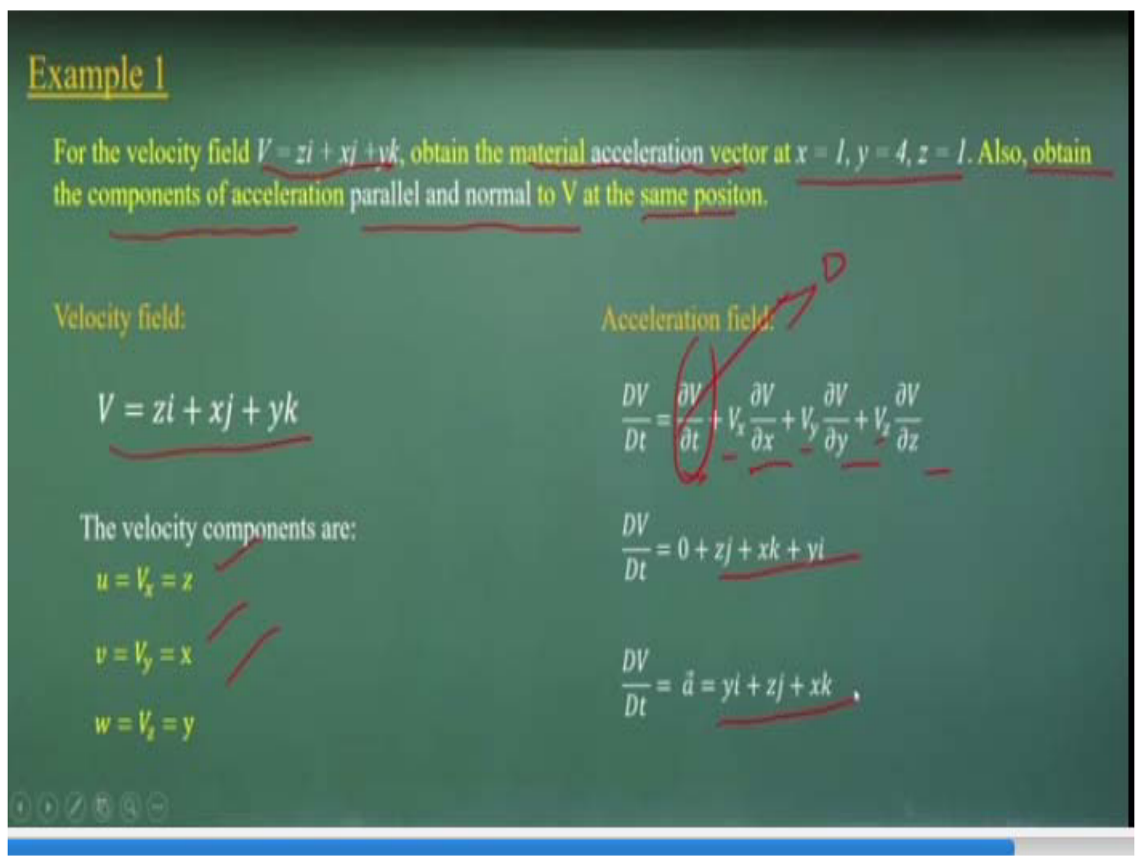

Now, let us solve this very few 2 examples problems; one is that the velocities field is given to us to compute the material accelerations factors okay, as we discuss it and the positions has given it, after that you obtain the component of acceleration is parallel normal to this V, okay so, let us solve the first problem is that, that means I know the velocity field, I know this u, v, w components, okay.

[For the velocity field $V = zi + xj + yk$, obtain the material acceleration vector at $x = 1, y = 4, z = 1$. Also, obtain the components of acceleration parallel and normal to V at the same positon]

Velocity field:

$$V = zi + xj + yk$$

The velocity components are:

$$u = V_x = z$$
$$v = V_y = \mathrm{x}$$
$$w = V_z = \mathrm{y}$$

Acceleration field:

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_x\frac{\partial V}{\partial x} + V_y\frac{\partial V}{\partial y} + V_z\frac{\partial V}{\partial z}$$

$$\frac{DV}{Dt} = 0 + zj + xk + yi$$

$$\frac{DV}{Dt} = \vec{a} = yi + zj + xk$$

I will do this partial derivative, substitute the Vx, Vy, Vz and the time derivative, there is no time components okay, so there is these components becomes 0, velocity does not have any time component that is become 0, so we just substitute it and you will get these accelerations as this one.

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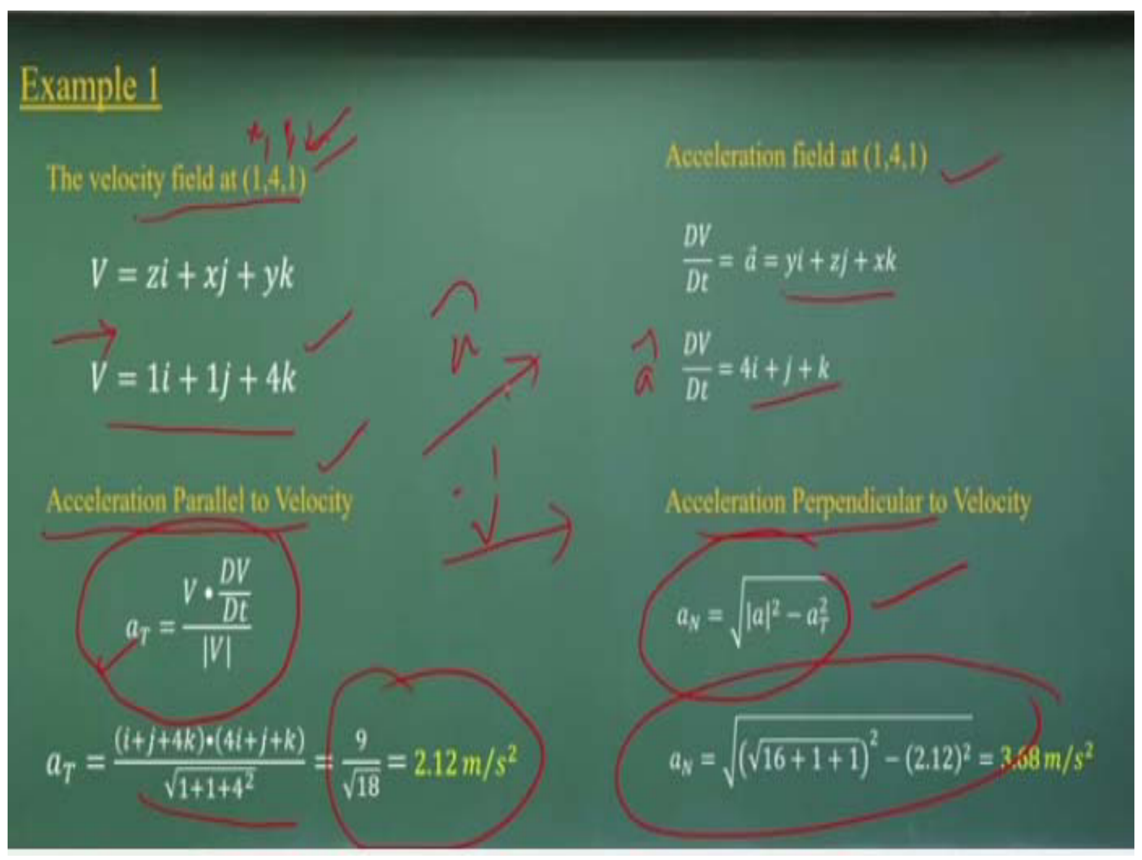

Then you substitute the; at the velocity field this 1, 4, 1, this x position, y and the z we just substitute it to get this velocity factors, the similar way we get this acceleration as we substitute the value. Now, what is telling that you find out the component of accelerations which is parallel to the velocity that means, we should do the dot product by magnitudes okay, you just try to understand the few vectors relationship, how to compute the component which is parallel to the velocity field.

The velocity field at (1,4,1)

$$V = zi + xj + yk$$

$$V = 1i + 1j + 4k$$

Acceleration Parallel to Velocity

$$a_T = \frac{V \bullet \frac{DV}{Dt}}{|V|}$$

$a_T = \frac{(i+j+4k)\bullet(4i+j+k)}{\sqrt{1+1+4^2}} = \frac{9}{\sqrt{18}} = 2.12\, m/s^2$

You will have this component and the acceleration perpendicular to these ones, I think you follow the any vector calculus book, you can find out how to; if I have a 2 vector velocity and acceleration vectors how to find out the accelerations which is a parallel to the velocity and what is the acceleration component parallel to this velocity okay, there is a 2 vectors now okay, one is acceleration factor and other is velocity vector.

Acceleration field at (1,4,1)

$$\frac{DV}{Dt} = \vec{a} = yi + zj + xk$$

$$\frac{DV}{Dt} = 4i + j + k$$

Acceleration Perpendicular to Velocity

$$a_N = \sqrt{|a|^2 - a_T^2}$$

$$a_N = \sqrt{\left(\sqrt{16+1+1}\right)^2 - (2.12)^2} = 3.68\, m/s^2$$

Next questions asking is that what is accelerations parallel to the velocity that the components what will get it here and what is that acceleration component; it is perpendicular to this that is what you just follow vector algebra, okay.

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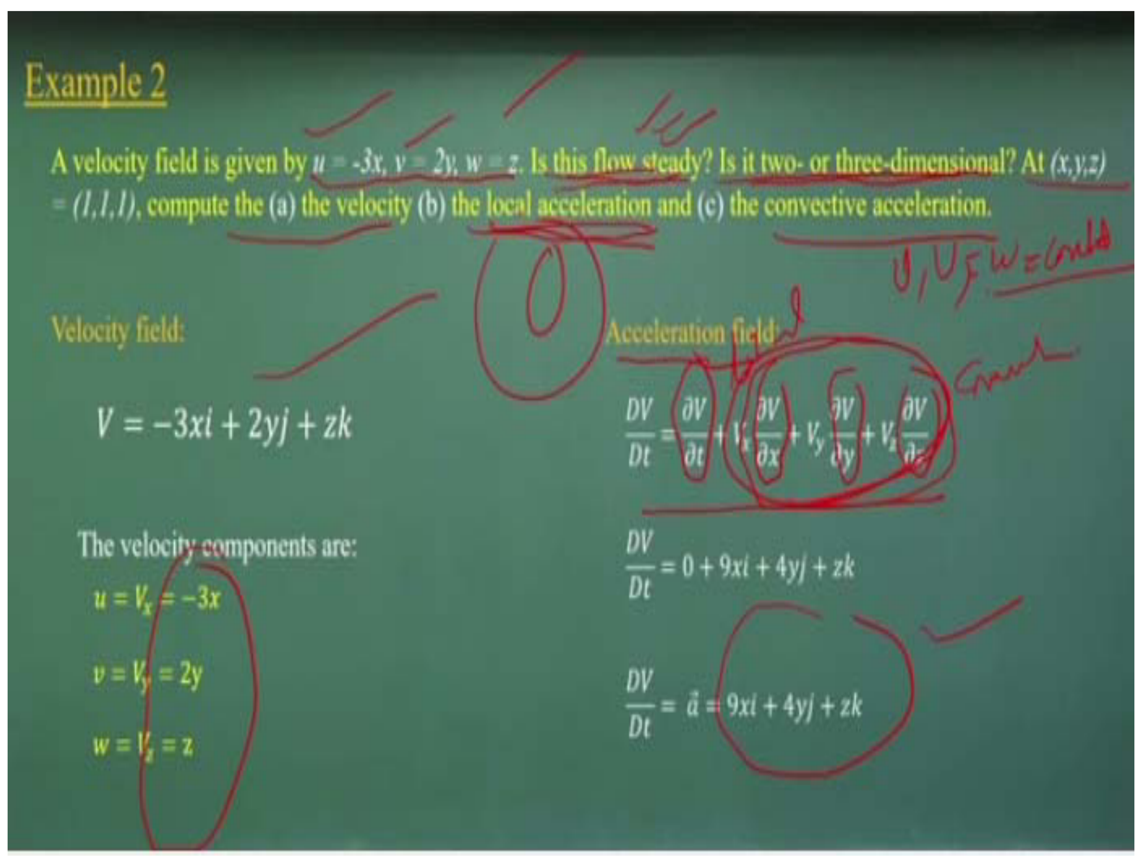

[A velocity field is given by *u = -3x, v = 2y, w = z*. Is this flow steady? Is it two- or three-dimensional? At *(x,y,z) = (1,1,1)*, compute the (a) the velocity (b) the local acceleration and (c) the convective acceleration.]

Now, talk about another very simple interesting problem that the velocity field is given to us okay, v, w, small v, u all it is given to us, is this flow steady, you can look it, there is no time components, so we can say this flow is steady, it is very straight forward. Is it a 2 dimensional, 3 dimensional; you just look it, there is a x component, y component, z component, so it is a 3 dimensional in terms of the positions.

Now, let x, y, z compute the velocity, which is very easy, local accelerations convective acceleration, so this is very easy problems as compared to previous ones, so you know the velocity field, you know these u, v, w components, you know these accelerations which this is part is local accelerations, this part is convective acceleration, you as we say these flow is steady without doing any calculations, we can say that local acceleration is 0.

Velocity field:

$$V = -3xi + 2yj + zk$$

The velocity components are:

$$u = V_x = -3x$$

$$v = V_y = 2\text{y}$$

$$w = V_z = \text{z}$$

Because there is no time component, if this flow is steady, the local acceleration becomes 0, we need not to do any calculations, with some common sense we can say that when you have the steady flow, local accelerations becomes 0 because your velocity field does not have a time component, it is independent of time. If it is that, there will be no local acceleration component, there will be convective accelerations component will be there.

So, what do we get it is a convective acceleration that is what we solve it and get these acceleration similarly, if you look at if velocity does not vary, these are all this gradient, if velocity does not vary in x, y, and z direction, my convective acceleration should be 0, so you just imagined it what is the velocity field is given to you, as I told earlier is that try to visualize the fluid flow by drawing these 3 dimensional figures in Matlab mathematica, any you have a lot of resources nowadays to have a 3 dimensional plots okay.

Even if in a Microsoft Excel, we can have a 3 dimensional plots, so making 3 dimensional plot and try to understand how do they varied, as I told you that when you compute the accelerations field, if the problems is lacking is the local acceleration is there or not, first look at whether the flow is steady or unsteady, if it is steady it independent to the time, there is no time component in u, v, w, we can say the steady flow.

Acceleration field:

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_x \frac{\partial V}{\partial x} + V_y \frac{\partial V}{\partial y} + V_z \frac{\partial V}{\partial z}$$

$$\frac{DV}{Dt} = 0 + 9xi + 4yj + zk$$

$$\frac{DV}{Dt} = \vec{a} = 9xi + 4yj + zk$$

And we can say that local acceleration should be 0, only it is a convective acceleration, similar way if my u, v, w all are constant, they do not vary with the time, all are constant, the convective acceleration becomes 0. So, you can visualize this thing, the problems of the fluid mechanics are too easy but you try to understand the functions how do they vary.

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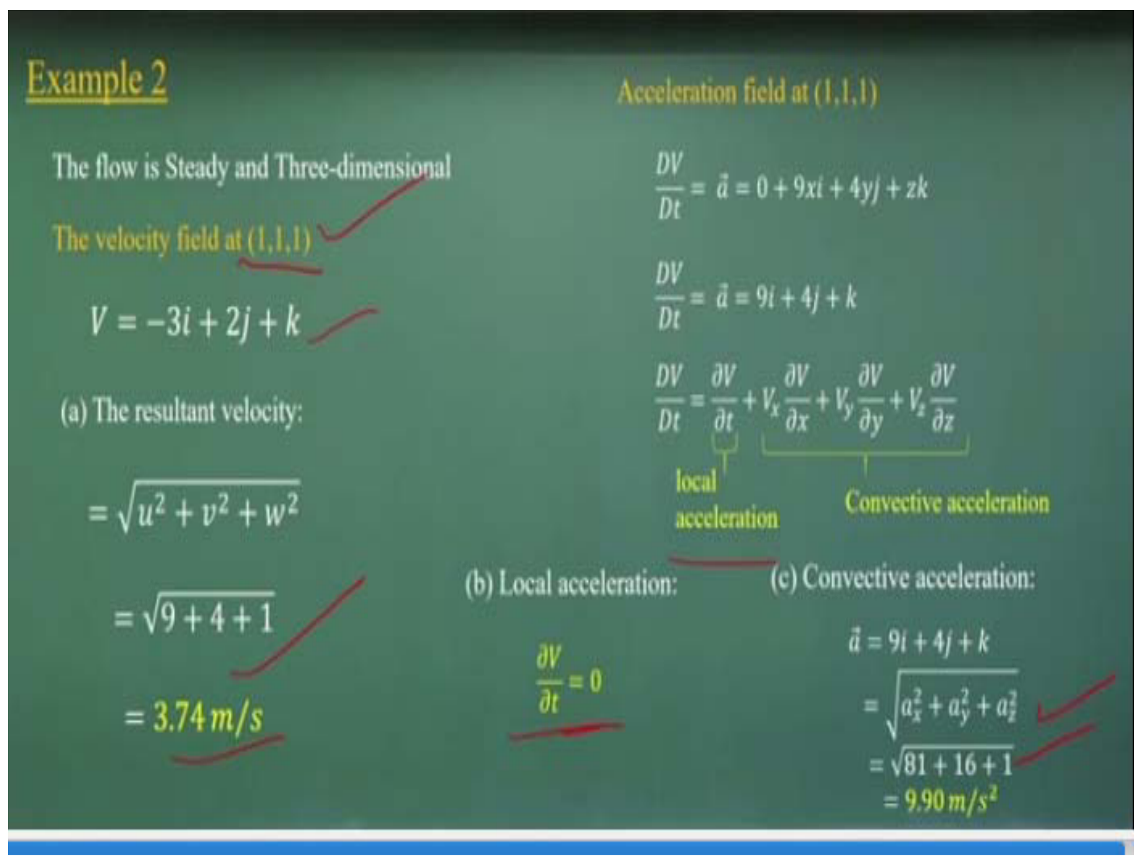

If you understand that we can use these simple mathematics to solve these things now, to compute the velocity field and acceleration field, we just substitute the velocity; at the positions 1 and 1, what should be that and what should be the resultant velocities as you know from vector calculations similar way, these local accelerations components also and convective accelerations component we can compute it.

The flow is Steady and Three-dimensional

The velocity field at (1,1,1)

$$V = -3i + 2j + k$$

(a) The resultant velocity:

$$= \sqrt{u^2 + v^2 + w^2}$$
$$= \sqrt{9 + 4 + 1}$$
$$= 3.74\, m/s$$

Acceleration field at (1,1,1)

$$\frac{DV}{Dt} = \vec{a} = 0 + 9xi + 4yj + zk$$
$$\frac{DV}{Dt} = \vec{a} = 9i + 4j + k$$
$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_x \frac{\partial V}{\partial x} + V_y \frac{\partial V}{\partial y} + V_z \frac{\partial V}{\partial z}$$

b) Local acceleration

$$\frac{\partial V}{\partial t} = 0$$

(c) Convective acceleration:

$$\vec{a} = 9i + 4j + k$$
$$= \sqrt{a_x^2 + a_y^2 + a_z^2}$$
$$= \sqrt{81 + 16 + 1}$$
$$= 9.90\, m/s^2$$

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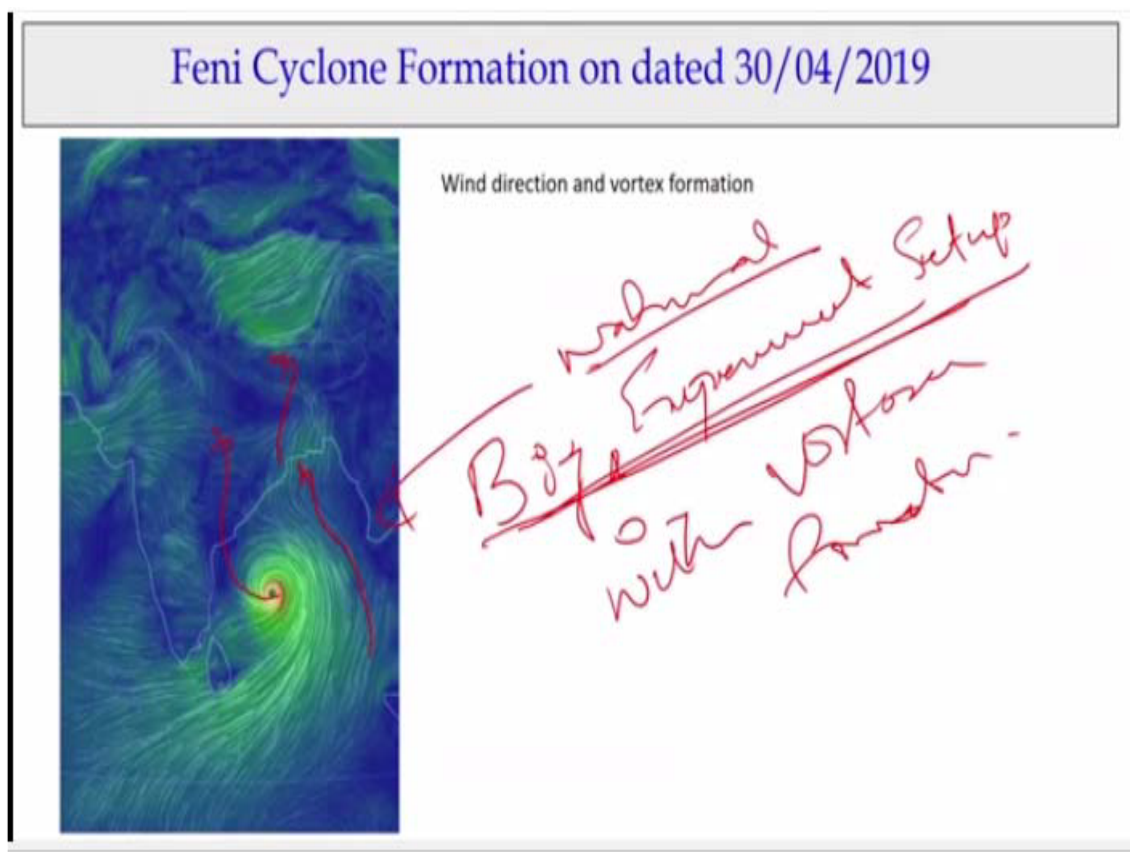

So, we can have a references this, with this let us conclude today lectures showing you very interesting photographs of today photographs which is available in Internet, today is 30th April

2019, okay you can see there was cyclone formations happened in Bay of Bengal wind directions and vortex formation, is it not very interesting; we talked about a small Hele-Shaw experiment set up, it is a big experiment set up, big, big is it correct; very big experiment set up of with vortex formation.

And these cyclones we are predicting it, we are tracking it what could be the cycle, you see these figures, you try to understand how complex flow patterns happens during the cyclone formations, how the vortex pendants, how the wind directions are changing it, so we have a big experiment, natural experiment setups is a cyclone formation, okay not this small experimental setup but the Hele-Shaw experiment setup with a colour dye experiment.

But todays we are fortunate enough to see the process at different scale, smaller scales to much, much bigger scale, with this let me finish these lectures with just showing you the summaries as I told it earlier.

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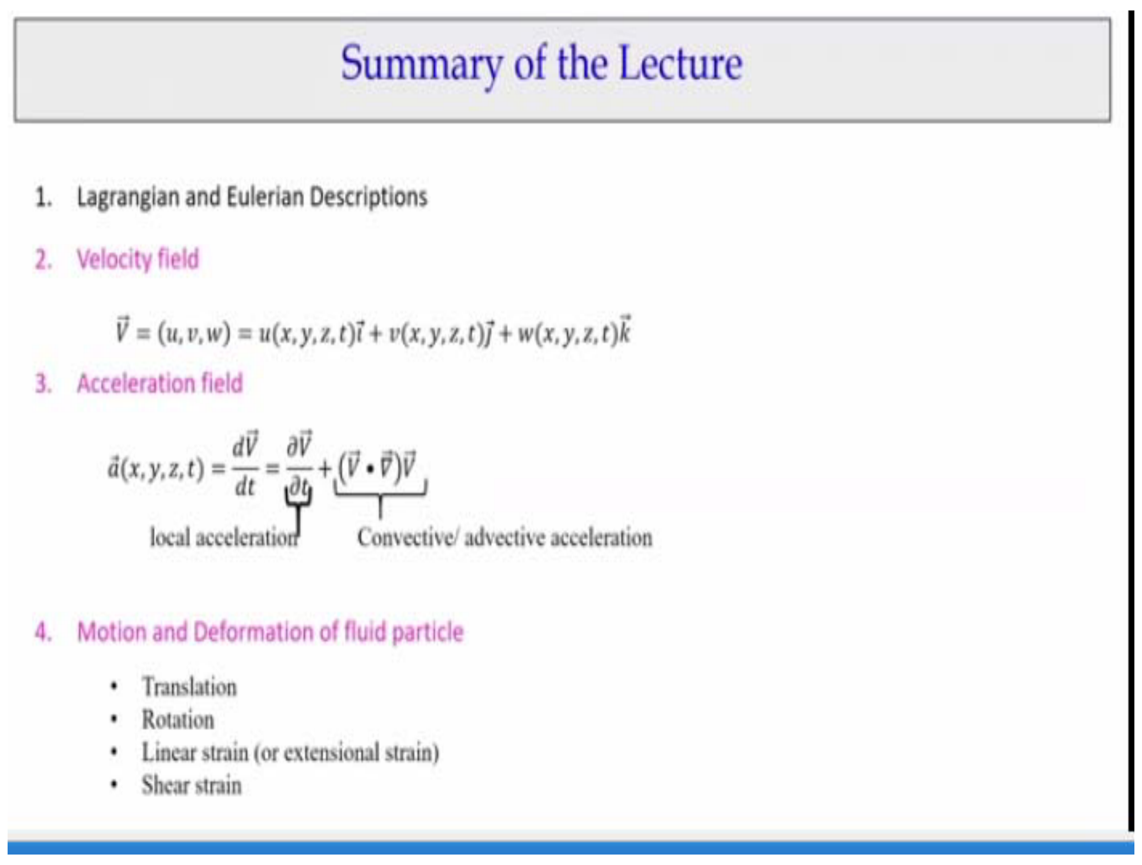

And today lecture, I just spend a lot of time you to just understand what is the difference between Lagrangian and the Eulerian descriptions and we talk about the velocity field and acceleration field.

1. Velocity field

$$\vec{V} = (u, v, w) = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

2. Acceleration field

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \bullet \vec{\nabla})\vec{V}$$

3. Motion and Deformation of fluid particle
    - Translation
    - Rotation
    - Linear strain (or extensional strain)
    - Shear strain

The next class we talk about more about the motion and deformations of fluid particles, vortex, vorticities, all we will discuss using virtual fluid balls, thank you a lot for this.